Broadcasting of three qubit entanglement via local copying and entanglement swapping

Satyabrata Adhikari*, B. S. Choudhury
Department of Mathematics,Bengal Engineering and Science University, Howrah-711103, West Bengal, India

February 1, 2008

Abstract

In this work, We investigate the problem of secretly broadcasting of three-qubit entangled state between two distant partners. The interesting feature of this problem is that starting from two particle entangled state shared between two distant partners we find that the action of local cloner on the qubits and the measurement on the machine state vector generates three-qubit entanglement between them. The broadcasting of entanglement is made secret by sending the measurement result secretly using cryptographic scheme based on orthogonal states. Further we show that this idea can be extended to generate three particle entangled state between three distant partners.

1 Introduction

No-cloning theorem is one of the most fundamental theorem in quantum computation and quantum information[1]. The theorem states that there does not exist any process, which turns two distinct nonorthogonal quantum states ψ,ϕ into states $\psi \otimes \psi$,$\phi \otimes \phi$

*Corresponding Author:Satyabrata Adhikari,E-Mail:satyyabrata@yahoo.com

respectively.This restrictions can be successfully utilized in quantum cryptography[2]. Although we cannot copy an unknown quantum state perfectly but one can always do it approximately. Beyond the no-cloning theorem, one can clone an arbitrary quantum state perfectly with some non-zero probability[3].In the past years,much progress has been made in designing quantum cloning machine. A first step towards the construction of approximate quantum cloning machine was taken by Buzek and Hillery in 1996 [4].They showed that the quality of the copies produced by their machine remain same for all input state.This machine is popularly known as universal quantum cloning machine (UQCM). Later this UQCM was proved to be optimal [5]. After that the different sets of quantum cloning machines like the set of universal quantum cloning machines, the set of state dependent quantum cloning machines (i.e. the quality of the copies depend on the input state) and the probabilistic quantum cloning machines were proposed. Entanglement[6], the heart of quantum information theory,play a crucial role in computational and communicational purposes. Therefore, as a valuable resource in quantum information processing, quantum entanglement has been widely used in quantum cryptography [7,19],quantum superdense coding [8] and quantum teleportation [9]. An astonishing feature of quantum information processing is that information can be "encoded" in non-local correlations between two separated particles. The more "pure" is the quantum entanglement, the more "valuable" is the given two-particle state. Therefore, to extract pure quantum entanglement from a partially entangled state, researchers had done lot of works in the past years on purification procedures[10].In other words, it is possible to compress locally an amount of quantum information.Now generally a question arises: whether the opposite is true or not i.e. can quantum correlations be "decompressed"? This question was tackled by several researchers [11,12] using the concept of "Broadcasting of quantum inseparability". Broadcasting is nothing but a local copying of non-local quantum correlations. That is the entanglement originally shared by a single pair is transferred into two less entangled pairs using only local operations. Suppose two distant parties A and B share two qubit entangled state

$$|\psi\rangle = \alpha|00\rangle_{AB} + \beta|11\rangle_{AB} \tag{1}$$

where α is real and β complex and the parameters satisfying the relation $\alpha^2 + |\beta|^2 = 1$. The first qubit belongs to A and the second belongs to B. Each of the two parties now perform local copier on their own qubit and then it turns out that for some values of α,

(1) non-local output states are inseparable, and

(2) local output states are separable.

In classical theory one can always broadcast information but in quantum theory, broadcasting is not always possible. H.Barnum et.al. showed that non-commuting mixed states cannot be broadcasted [16]. However for pure states broadcasting is equivalent to cloning.

V.Buzek et.al. were the first who showed that the decompression of initial quantum entanglement is possible, i.e. that from a pair of entangled particles, two less entangled pairs can be obtained by local operation. That means inseparability of quantum states can be partially broadcasted (cloned) with the help of local operation. They used optimal universal quantum cloners for local copying of the subsystems and showed that the non-local outputs are inseparable if

$$\frac{1}{2} - \frac{\sqrt{39}}{16} \leq \alpha^2 \leq \frac{1}{2} + \frac{\sqrt{39}}{16} \tag{2}$$

Further S.Bandyopadhyay et.al. [12] studied the broadcasting of entanglement and showed that only those universal quantum cloners whose fidelity is greater than $\frac{1}{2}(1 + \sqrt{\frac{1}{3}})$ are suitable because only then the non-local output states becomes inseparable for some values of the input parameter α. They proved that an entanglement is optimally broadcast only when optimal quantum cloners are used for local copying and also showed that broadcasting of entanglement into more than two entangled pairs is not possible using only local operations. I.Ghiu investigated the broadcasting of entanglement by using local $1 \rightarrow 2$ optimal universal asymmetric Pauli machines and showed that the inseparability is optimally broadcast when symmetric cloners are applied [21].

Motivated from the previous works on broadcasting of entanglement, we investigate the problem of secretly broadcasting of three-qubit entangled state between two distant partners with universal quantum cloning machine and then the result is generalized to generate secret entanglement among three parties. Three-qubit entanglement between two distant partners can be generated as follows: Let us suppose that the two distant partners share an entangled state $|\psi\rangle_{12} = \alpha|00\rangle + \beta|11\rangle$. The two parties then apply optimal universal quantum cloning machine on their respective qubits to produce four qubit state $|\chi\rangle_{1234}$. One party (say, Alice) then performs measurement on her quantum cloning machine state vectors. After that she inform Bob about her measurement result using Goldenberg and Vaidman's quantum cryptographic scheme based on orthogonal states. Getting measurement result from Alice, other partner (say, Bob) also performs measurement on his quantum cloning machine state vectors and using the same cryptographic scheme, he sends his measurement outcome to Alice. Since the measurement results are interchanged secretly so Alice and Bob share secretly four qubit state. They again apply the cloning machine on one of their respective qubits and generate six qubit state $|\phi\rangle_{125346}$. Therefore, each parties have three qubit each. Among six qubit state, we interestingly find that there exists two three qubit state shared by Alice and Bob which are entangled for some values of the input parameter α^2.

In the second part, we investigate the problem of secret entanglement broadcasting among three distant parties. To solve this problem, we start with the result of the first part i.e. we assume that the two distant partners (say, Alice and Bob) shared a three qubit entangled state. Without any loss of generality, we assume that among three qubits, two are with Alice and one with Bob. Then Alice teleport one of the qubit to the third distant partner (say, Carol). After the completion of the teleportation procedure, we find that the three distant partners shared a three qubit entangled state for the same values of the input parameters α^2 as in the first part of the protocol.

In broadcasting of inseparability, we generally use Peres-Horodecki criteria to show the inseparability of non-local outputs and separability of local outputs.

Peres-Horodecki Theorem [13,14]: The necessary and sufficient condition for the state $\hat{\rho}$ of two spins $\frac{1}{2}$ to be inseparable is that at least one of the eigen values of the partially transposed operator defined as $\rho^T_{m\mu,n\nu} = \rho_{m\nu,n\mu}$ is negative. This is equivalent to the condition that at least one of the two determinants

$$W_3 = \begin{vmatrix} \rho_{00,00} & \rho_{01,00} & \rho_{00,10} \\ \rho_{00,01} & \rho_{01,01} & \rho_{00,11} \\ \rho_{10,00} & \rho_{11,00} & \rho_{10,10} \end{vmatrix} \text{ and } W_4 = \begin{vmatrix} \rho_{00,00} & \rho_{01,00} & \rho_{00,10} & \rho_{01,10} \\ \rho_{00,01} & \rho_{01,01} & \rho_{00,11} & \rho_{01,11} \\ \rho_{10,00} & \rho_{11,00} & \rho_{10,10} & \rho_{11,10} \\ \rho_{10,01} & \rho_{11,01} & \rho_{10,11} & \rho_{11,11} \end{vmatrix}$$

is negative.

For the security of the broadcasting of entanglement, we use L.Goldenberg et.al. quantum cryptographic scheme which was based on orthogonal states [15]. The cryptographic scheme is described by Figure-I.

All the previous works on the broadcasting of entanglement deals with the generation of two 2-qubit entangled state starting from a 2-qubit entangled state using either optimal universal symmetric cloner [4,5] or asymmetric cloner [24,25]. The generated two qubit entangled state can be used as a quantum channel in quantum cryptography, quantum teleportation etc. The advantage of our protocol over other protocols of broadcasting is that we are able to provide a protocol which generates secret quantum channel between distant partners. The introduced protocol generate two 3-qubit entangled state between two distant partners starting from a 2-qubit entangled state and also provide the security of the generated quantum channel. Not only that we also generalize our protocol from two parties to three parties and show that the generated 3-qubit entangled states can serve as a secured quantum channel between three parties. Now to hack the quantum information, hackers have to do two things: First, they have to gather knowledge about the initially shared entangled state and secondly, they have to collect information about the measurement result performed by two distant partners. Therefore, the quantum

channel generated by our protocol is more secured and hence can be used in various protocols viz. quantum key distribution protocols [22,23].

We then distribute our work in the remaining three sections. In section 2, we present our idea with a specific example for broadcasting of three-qubit entangled state shared between two distant partners. In section 3, we generalize this idea to generate three-qubit entangled state shared between three distant parties. To implement the idea, we use the concept of entanglement swapping. The last section is devoted to the conclusion.

2 Secretly broadcasting of 3-qubit entangled state between two distant partners

In this section, firstly we define broadcasting of three qubit entanglement, open entanglement and close entanglement.

Let the previously shared entangled state (1) described by the two qubit density operator be ρ_{13}. Using B-H quantum cloning machine twice by the distant partners (Alice and Bob) on their respective qubits, they generate total six-qubit state ρ_{125346} between them. Therefore, Alice has three qubits '1','2' and '5' and Bob possesses three qubits '3', '4' and '6'.

Definition-1: The three-qubit entanglement is said to be broadcast if (i) Any of the two local outputs (say (ρ_{12},ρ_{15}) in Alice's side and (ρ_{34},ρ_{36}) in Bob's side) are separable (ii) One local output (say ρ_{25} in Alice's side and ρ_{46} in Bob's side) is inseparable and associated with these local inseparable output,two non-local outputs (say (ρ_{23},ρ_{35}) and (ρ_{14},ρ_{16})) are inseparable.

Definition-2: An entanglement is said to be closed if each party has non-local correlation with other parties. For instance, any three particle entangled state described by the density operator ρ_{325} is closed if ρ_{32},ρ_{25} and ρ_{35} are entangled state. Otherwise,

it is said to be an open entanglement.

Closed entanglement and open entanglement is shown in Figure-VI and Figure-VII respectively.

Now we are in a position to discuss our protocol for secretly broadcasting of three qubit entangled state. we start the protocol with two qubit entangled state $|\psi\rangle_{13}$ shared between two distant partners popularly known as Alice and Bob. Particles '1' and '3' possessed by Alice and Bob respectively. Alice and Bob then operates quantum cloning machine on their respective qubits. After cloning procedure,Alice perform measurement on the quantum cloning machine state vector and send the measurement result to Bob. After getting measurement result from Alice; Bob perform measurement on his quantum cloning machine state vector and send the measurement result to Alice. Consequently, the two distant partners share a four qubit state $|\zeta\rangle_{1234}$. Now Alice has two qubits '1' and '2' and Bob '3' and '4' respectively. Both of them again operates quantum cloning machine on one of the qubits, they possess. As a result, the distant parties now share six qubit state $|\phi\rangle_{125346}$ in which three qubits '1','2' and '5'possessed by Alice and remaining three qubits '3','4' and '6' possessed by Bob. Now if there exists two 3-qubit entangled state between two distant partners for some values of the input parameter α^2, then only we can secretly broadcast 3-qubit entangled state using only universal quantum cloning machine. The word 'secretly' is justified by observing an important fact that the transmission of measurement result from Alice to Bob and Bob to Alice has been done by using Goldenberg and Vaidman's quantum cryptographic scheme. Therefore, message regarding measurement results can be transmitted secretly between two distant partners. Hence, the broadcasted three-qubit entangled state is only known to Alice and Bob and not to the third party 'Eve'. As a result, these newly generated three-qubit entangled states can be used as a secret quantum channel in various quantum cryptographic scheme.

Now to understand our protocol more clearly, we again discuss the whole protocol below

by considering a specific example.

Step -1

Let the two particle entangled state shared by two distant partners Alice and Bob is given by

$$|\psi\rangle_{13} = \alpha|00\rangle + \beta|11\rangle \tag{3}$$

where α is real and β is complex with $\alpha^2 + |\beta|^2 = 1$. This initial entangled state is shown in Figure-II.

Step-2

The B-H quantum copier is given by the transformation

$$|0\rangle|\rangle|Q\rangle \rightarrow \sqrt{\frac{2}{3}}|00\rangle|Q_0\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_1\rangle \tag{4}$$

$$|1\rangle|\rangle|Q\rangle \rightarrow \sqrt{\frac{2}{3}}|11\rangle|Q_1\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_0\rangle \tag{5}$$

where $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle$ and $|Q_0\rangle, |Q_1\rangle$ are orthogonal quantum cloning machine state vectors.

Alice and Bob then operates B-H quantum cloning machine locally to copy the state of their respective particles. Therefore, after operating quantum cloning machine, Alice and Bob both of them are able to approximate clone the state of the particle and consequently the combined system of four qubits is given by

$$|\chi\rangle_{1234} = [(\frac{2\alpha}{3}|0000\rangle + \frac{\beta}{3}|\psi^+\rangle|\psi^+\rangle)|Q_0\rangle^B + (\frac{\sqrt{2}\alpha}{3}|00\rangle|\psi^+\rangle + \frac{\sqrt{2}\beta}{3}|\psi^+\rangle|11\rangle) |Q_1\rangle^B]|Q_0\rangle^A + [(\frac{\sqrt{2}\alpha}{3}|\psi^+\rangle|00\rangle + \frac{\sqrt{2}\beta}{3}|11\rangle|\psi^+\rangle)|Q_0\rangle^B + (\frac{\alpha}{3}|\psi^+\rangle|\psi^+\rangle + \frac{2\beta}{3}|1111\rangle)|Q_1\rangle^B]|Q_1\rangle^A \tag{6}$$

The subscript 1,2 and 3,4 refers to two approximate copy qubits in the Alice's and Bob's side respectively. Also $|\rangle^A$ and $|\rangle^B$ denotes quantum cloning machine state vectors in Alice's and Bob's side respectively. This fact is explained by Figure-III.
Alice then performs measurement on the quantum cloning machine state vectors in the basis $\{|Q_0\rangle^A, |Q_1\rangle^A\}$. Thereafter, Alice inform Bob about her measurement result using Goldenberg and Vaidman's quantum cryptographic scheme based on orthogonal states which is discussed in the previous section. After getting measurement result from Alice, Bob also performs measurement on the quantum cloning machine state vectors in the basis $\{|Q_0\rangle^B, |Q_1\rangle^B\}$ and then using the same cryptographic scheme, he sends his measurement outcome to Alice. In this way Alice and Bob interchange their measurement results secretly.

Step-3

After measurement, let the state shared by Alice and Bob is given by

$$|\zeta_a\rangle_{1234} = \frac{1}{\sqrt{N}}[\frac{2\alpha}{3}|0000\rangle + \frac{\beta}{3}|\psi^+\rangle|\psi^+\rangle] \tag{7}$$

Where $N = \frac{3\alpha^2+1}{9}$ represents the normalization factor.
Afterward, Alice and Bob again operates their respective cloners on the qubits '2' and '4' respectively and therefore, the total state of six qubits is given by

$$|\phi\rangle_{125346} = \frac{1}{\sqrt{N}}[\frac{2\alpha}{3}[|0\rangle_1 \otimes (\sqrt{\frac{2}{3}}|00\rangle|Q_0\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_1\rangle)_{25} \otimes |0\rangle_3 \otimes (\sqrt{\frac{2}{3}}|00\rangle|Q_0\rangle$$
$$+\frac{1}{\sqrt{3}}|\psi^+\rangle|Q_1\rangle)_{46} + \frac{\beta}{6}[|0\rangle_1 \otimes (\sqrt{\frac{2}{3}}|11\rangle|Q_1\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_0\rangle)_{25} \otimes |0\rangle_3 \otimes$$
$$(\sqrt{\frac{2}{3}}|11\rangle|Q_1\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_0\rangle)_{46} + |0\rangle_1 \otimes (\sqrt{\frac{2}{3}}|11\rangle|Q_1\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_0\rangle)_{25} \otimes |1\rangle_3 \otimes$$
$$(\sqrt{\frac{2}{3}}|00\rangle|Q_0\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_1\rangle)_{46} + |1\rangle_1 \otimes (\sqrt{\frac{2}{3}}|00\rangle|Q_0\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_1\rangle)_{25} \otimes |0\rangle_3 \otimes$$
$$(\sqrt{\frac{2}{3}}|11\rangle|Q_1\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_0\rangle)_{46} + |1\rangle_1 \otimes (\sqrt{\frac{2}{3}}|00\rangle|Q_0\rangle + \frac{1}{\sqrt{3}}|\psi^+\rangle|Q_1\rangle)_{25} \otimes |1\rangle_3 \otimes$$

$$(\sqrt{\frac{2}{3}}|00\rangle|Q_0\rangle+\frac{1}{\sqrt{3}}|\psi^+\rangle|Q_1\rangle)_{46}] \tag{8}$$

Now our task is to see whether we can generate two 3-qubit entangled state from above six qubit state or not. To examine the above fact, we have to consider two 3-qubit state described by the density operators ρ_{146} and ρ_{325}. To understand more clearly, see figure-IV.

The density operator ρ_{146} is given by

$$\rho_{146}=\frac{1}{N}[\frac{4\alpha^2}{9}(\frac{2}{3}|000\rangle\langle000|+\frac{1}{3}|0\psi^+\rangle\langle0\psi^+|)+\frac{\alpha\beta^*}{9}(\frac{\sqrt{2}}{3}|000\rangle\langle1\psi^+|+ \\ \frac{\sqrt{2}}{3}|0\psi^+\rangle\langle111|)+\frac{\alpha\beta}{9}(\frac{\sqrt{2}}{3}|111\rangle\langle0\psi^+|+\frac{\sqrt{2}}{3}|1\psi^+\rangle\langle000|)+\frac{|\beta|^2}{36}(\frac{2}{3}|011\rangle\langle011|+ \\ \frac{2}{3}|0\psi^+\rangle\langle0\psi^+|+\frac{2}{3}|000\rangle\langle000|+\frac{2}{3}|111\rangle\langle111|+\frac{2}{3}|1\psi^+\rangle\langle1\psi^+|+\frac{2}{3}|100\rangle\langle100|)] \tag{9}$$

The density operator ρ_{325} describes the other three qubit state looks exactly the same as ρ_{146}.

Now to show the state described by the density operator ρ_{146} is entangled, we have to show that the two qubit states described by the density operators ρ_{14},ρ_{16} and ρ_{46} are entangled i.e. we have to show that there exist some values of the input state parameter α^2 for which the three-qubit state is a closed entangled state.

The reduced density operators ρ_{14},ρ_{16} and ρ_{46} are given by

$$\rho_{16}=\rho_{14}=\frac{1}{N}[\frac{4\alpha^2}{9}(\frac{5}{6}|00\rangle\langle00|+\frac{1}{6}|01\rangle\langle01|)+\frac{2\alpha\beta^*}{27}|00\rangle\langle11|+\frac{2\alpha\beta}{27}|11\rangle\langle00|+ \\ \frac{|\beta|^2}{36}(|00\rangle\langle00|+|01\rangle\langle01|+|10\rangle\langle10|+|11\rangle\langle11|)] \tag{10}$$

$$\rho_{46}=\frac{1}{N}[\frac{4\alpha^2}{9}(\frac{2}{3}|00\rangle\langle00|+\frac{1}{6}(|01\rangle\langle01|+|01\rangle\langle10|+|10\rangle\langle01|+|10\rangle\langle10|))+ \\ \frac{|\beta|^2}{36}(\frac{4}{3}|00\rangle\langle00|+\frac{4}{3}|11\rangle\langle11|+\frac{2}{3}(|01\rangle\langle01|+|01\rangle\langle10|+|10\rangle\langle01|+|10\rangle\langle10|))] \tag{11}$$

Now using Peres-Horodecki theorem, we find that the state described by the density operators ρ_{16} and ρ_{14} are entangled if $0.18<\alpha^2<1$ and the state described by the

density operator ρ_{46} is entangled if $0.61 < \alpha^2 < 1$. Therefore, we can say that the state described by the density operator ρ_{146} is a closed three qubit entangled state if $0.61 < \alpha^2 < 1$. Similarly, the other reduced density operator ρ_{325} describe a closed entangled state if $0.61 < \alpha^2 < 1$.

Also the other two-qubit state described by the density operators ρ_{12},ρ_{15},ρ_{34} and ρ_{36} are given by

$$\rho_{12} = \rho_{15} = \rho_{34} = \rho_{36} = \frac{1}{N}[\frac{4\alpha^2}{9}(\frac{5}{6}|00\rangle\langle 00| + \frac{1}{6}|01\rangle\langle 01|) + \frac{|\beta|^2}{36}(\frac{1}{3}|00\rangle\langle 00| + \frac{5}{3}|01\rangle\langle 01| + \frac{4}{3}|01\rangle\langle 10| + \frac{4}{3}|10\rangle\langle 01| + \frac{5}{3}|10\rangle\langle 10| + \frac{1}{3}|11\rangle\langle 11|)] \quad (12)$$

These density operators are separable only when $0.27 < \alpha^2 < 1$. Hence, broadcasting of three-qubit entangled state is possible when $0.61 < \alpha^2 < 1$.

Now, our task is to find out how is the entanglement distributed over the state i.e. how much are the two qubit density operators ρ_{16}, ρ_{14} and ρ_{46} are entangled. To evaluate the amount of entanglement,We have to calculate the concurrence defined by Wootters [20] and hence entanglement of formation.

Wootters gave out, for the mixed state $\hat{\rho}$ of two qubits, the concurrence is

$$C = max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0) \quad (13)$$

where the λ_i, in decreasing order, are the square roots of the eigen values of the matrix $\rho^{\frac{1}{2}}(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\rho^{\frac{1}{2}}$ and ρ^* denotes the complex conjugation of ρ in the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle$ and σ_y is the Pauli operator.

The entanglement of formation E_F can then be expressed as a function of C, namely

$$E_F = -\frac{1+\sqrt{1-C^2}}{2} \quad log_2\frac{1+\sqrt{1-C^2}}{2} - \frac{1-\sqrt{1-C^2}}{2} \quad log_2\frac{1-\sqrt{1-C^2}}{2} \quad (14)$$

After a little bit calculation, we find that the concurrence and hence the entanglement of formation depends on the probability α^2. Therefore, we have to calculate the amount

of entanglement in the 2-qubit states described by the reduced density operators ρ_{16}, ρ_{14} and ρ_{46} in the range $0.61 < \alpha^2 < 1$ because the two qubit reduced density operators are entangled in this range of the input state parameter α^2. Since concurrence depends on α^2 so it varies as α^2 varies. Therefore, when $0.61 < \alpha^2 < 1$, the concurrences for the mixed states described by density operators ρ_{16}, ρ_{14} varies from 0.17 to 0.29 while the concurrence for the mixed states described by density operators ρ_{46} varies from 0.08 to 0.15 respectively. Using the relation (16) and the values of concurrence, we find that the entanglement of formation for the density oprators ρ_{16}, ρ_{14} varies from 0.06 to 0.15 while the entanglement of formation for the density operator ρ_{46} varies from 0.01 to 0.03 respectively. Therefore, the generated three-qubit entangled state is a weak closed entangled state in the sense that the amount of entanglement in the two-qubit density operators are very low. Further, the above results shows that the entanglement between the qubits 1 and 6 (1 and 4) is higher than between the the qubits 4 and 6.

Furthermore, if the measurement results are either $\frac{\sqrt{2}\alpha}{3}|00\rangle|\psi^{+}\rangle + \frac{\sqrt{2}\beta}{3}|\psi^{+}\rangle|11\rangle$ or $\frac{\sqrt{2}\alpha}{3}|\psi^{+}\rangle|00\rangle + \frac{\sqrt{2}\beta}{3}|11\rangle|\psi^{+}\rangle$, then the two 3-qubit state described by the density operators ρ_{146} and ρ_{325} are different and the broadcasting is possible for $0.6 < \alpha^2 < 1$ or $0.14 < \alpha^2 < 0.4$ according to the outcomes. Also if the outcome of the measurement is $\frac{\alpha}{3}|\psi^{+}\rangle|\psi^{+}\rangle + \frac{2\beta}{3}|1111\rangle$, then the state described by the density operators ρ_{146} and ρ_{325} are identical and the broadcasting is possible for $0.38 < \alpha^2 < 0.73$.

3 Secretly generation of two 3-qubit entangled state between three distant partners

In this section, we attempt to answer a question: can we secretly generate two 3-qubit entangled state shared between three distant partners using LOCC? The answer is in affirmative. Now we show below that the 3-qubit entangled state shared between three

distant partners can be generated by two different processes.

To generate three-qubit entangled state between three distant partners, we require only two well-known concept: (i) quantum cloning and (ii) entanglement swapping

Entanglement swapping [17,18] is a method that enables one to entangle two quantum systems that do not have direct interaction with one another. S.Bose et.al. [17] generalized the procedure of entanglement swapping and obtained a scheme for manipulating entanglement in multiparticle systems. They showed that this scheme can be regarded as a method of generating entangled states of many particles. An explicit scheme that generalizes entanglement swapping to the case of generating a 3-particle GHZ state from three Bell pairs has been presented by Zukowski et.al. The standard entanglement swapping helps to save a significant amount of time when one wants to supply two distant users with a pair of atoms or electrons (or any particle possessing mass) in a Bell state from some central source. The entanglement swapping can be used, with some probability which we quantify, to correct amplitude errors that might develop in maximally entangled states during propagation. In this work, we use the concept of entanglement swapping in the generation of three-qubit entanglement between three distant partners.

Now we are in a position to discuss the protocol for secretly generation of two 3-qubit entangled state between three distant partners via quantum cloning and entanglement swapping.

Let us suppose for the implementation of any particular cryptographic scheme, three distant partners Alice, Bob and Carol wants to generate two three qubit entangled state between them. To do the same task, let us assume that initially Alice-Bob and Carol-Alice share two qubit entangled states described by the density operators ρ_{13}, ρ_{78}, where Alice has qubits '1' and '8', Bob and Carol possess qubit '3' and '7' respectively. Then Alice and Bob adopting the broadcasting process described in the previous section to

generate two three-qubit entangled state in between them. Therefore, Alice and Bob now have two 3-qubit entangled state described by the density operators ρ_{146} and ρ_{325} where Alice has qubits '1','2'and '5' and Bob possesses '3','4' and '6'. Now we are in a position for the illustration of the generation of 3-qubit entangled between three parties at distant places by using the concept of entanglement swapping.

Without any loss of generality, we take a three-qubit entangled state between two distant parties described by the density operator ρ_{325}.

The density operator ρ_{325} can be rewritten as

$$\rho_{325} = \frac{1}{N}[\frac{4\alpha^2}{9}(\frac{2}{3}|000\rangle\langle 000| + \frac{1}{3}|0\psi^+\rangle\langle 0\psi^+|) + \frac{\alpha\beta^*}{9}(\frac{\sqrt{2}}{3}|000\rangle\langle 1\psi^+| + \frac{\sqrt{2}}{3}|0\psi^+\rangle\langle 111|)$$
$$+\frac{\alpha\beta}{9}(\frac{\sqrt{2}}{3}|111\rangle\langle 0\psi^+| + \frac{\sqrt{2}}{3}|1\psi^+\rangle\langle 000|) + \frac{|\beta|^2}{36}(\frac{2}{3}|011\rangle\langle 011| + \frac{2}{3}|0\psi^+\rangle\langle 0\psi^+| +$$
$$\frac{2}{3}|000\rangle\langle 000| + \frac{2}{3}|111\rangle\langle 111| + \frac{2}{3}|1\psi^+\rangle\langle 1\psi^+| + \frac{2}{3}|100\rangle\langle 100|)] \tag{15}$$

where qubits 2 and 5 possessed by Alice and qubit 3 possessed by Bob respectively.

To achieve the goal of the generation of three qubit entangled state between three distant partners, we proceed in the following way:

Let Alice and Carol shared a singlet state

$$|\psi^-\rangle_{87} = (\frac{1}{\sqrt{2}})(|01\rangle - |10\rangle) \tag{16}$$

where particles 8 and 7 possessed by Alice and Carol respectively.

The combined state between Alice,Bob and Carol is given by the

$$\rho_{32587} = \rho_{325} \otimes |\psi^-\rangle_{78}\langle\psi^-| \tag{17}$$

Alice then perform Bell state measurement on the particles 2 and 8 in the basis $\{|B_1^{\pm}\rangle, |B_2^{\pm}\rangle\}$, where $|B_1^{\pm}\rangle = (\frac{1}{\sqrt{2}})(|00\rangle \pm |11\rangle)$, $|B_2^{\pm}\rangle = (\frac{1}{\sqrt{2}})(|01\rangle \pm |10\rangle)$

If the measurement result is $|B_1^+\rangle$, then the 3-qubit density operator is given by

$$\rho_{357} = \frac{1}{N}[\frac{4\alpha^2}{9}[\frac{2}{3}|001\rangle\langle 001| + \frac{1}{6}(|011\rangle\langle 011| - |011\rangle\langle 000| - |000\rangle\langle 011| + |000\rangle\langle 000|)] +$$
$$\frac{\alpha\beta^*}{27}(|001\rangle\langle 111| - |001\rangle\langle 100| + |000\rangle\langle 110| - |011\rangle\langle 110|) + \frac{\alpha\beta}{27}(-|110\rangle\langle 011| + |110\rangle\langle 000|$$
$$+|111\rangle\langle 001| - |100\rangle\langle 001|) + \frac{|\beta|^2}{36}[\frac{2}{3}(|010\rangle\langle 010| + |001\rangle\langle 001| + |110\rangle\langle 110| + |101\rangle\langle 101|)$$
$$+\frac{1}{3}(|011\rangle\langle 011| - |011\rangle\langle 000| - |000\rangle\langle 011| + |000\rangle\langle 000| + |111\rangle\langle 111| - |111\rangle\langle 100| -$$
$$|100\rangle\langle 111| + |100\rangle\langle 100|)] \tag{18}$$

After Bell-state measurement, Alice announces publicly the measurement result. Thereafter, Alice,Bob and Carol operate an unitary operator $U_1 = I_3 \otimes (\sigma_z)_5 \otimes (\sigma_x)_7$ on their respective particles to retrieve the state described by the density operator ρ_{325}.
If the measurement result is $|B_1^-\rangle$ or $|B_2^+\rangle$ or $|B_2^-\rangle$ then accordingly they operate an unitary operator $U_2 = I_3 \otimes (I_5) \otimes (\sigma_x)_7$ or $U_3 = I_3 \otimes (I_5) \otimes (\sigma_z)_7$ or $U_4 = I_3 \otimes (I_3) \otimes (I_7)$ on their respective particles to retrieve the state described by the density operator ρ_{325}.
Hence, we find that after getting the measurement result, each party (Alice, Bob and Carol) apply the suitable unitary operator on their respective particles to share the 3-qubit entangled state in between them,which is previously shared between only two distant partners Alice and Bob.
Also we note an important fact that the generated 3-qubit entangled state is totally secret between three distant partners because the outcome of the measurement on the machine state vector is totally unknown to the eavesdropper. Furthermore, the reduced density operator describing 3-qubit state between two distant partners and the reduced density operator describing 3-qubit state between three distant partners are entangled for the same range of α^2.

We can understand the above protocol pictorially (Figure-V and Figure-VI):
Figure-V: Alice and Bob share a 3-qubit entangled state described by the density operator ρ_{325}. Alice and Carol share a singlet state described by the density operator $\rho_{78} = |\psi^-\rangle_{78}\langle\psi^-|$. Then Alice perform Bell-state measurement (BSM) on particles 2 and 8 of the joint state described by the density operator $\rho_{325} \otimes \rho_{78}$.

Figure-VI: Finally, after applying suitable unitary operators, 3-qubit entangled state described by the density operator ρ_{325} is generated between three distant partners Alice, carol and Bob.

Therefore, in this section we describe the secretly generation of 3-qubit entangled state between three distant partners starting from 3-qubit entangled state shared between two distant partners using quantum cloning and entanglement swapping. This quantum channel generated by the above procedures can be regarded as a secret quantum channel because the result of the measurement on the machine state vectors transmitted secretly by quantum cryptographic scheme.

4 Conclusion

In this work, we present a protocol for the secret broadcasting of three-qubit entangled state between two distant partners. Here we should note an important fact that the two copies of three-qubit entangled state is not generated from previously shared three-qubit entangled state but from previously shared two-qubit entangled state using quantum cloning machine. They send their measurement result secretly using cryptographic scheme so that the produced copies of the three-qubit entangled state shared between two distant parties can serve as a secret quantum channel. We also extend this idea to create three-particle entangled state secretly between three distant partners using quantum cloning and entanglement swapping procedure.

Acknowledgement

S.A wishes to acknowledge the support given by CSIR project F.No.8/3(38)/2003-EMR-1, New Delhi. S.A would like to thank Samir Kunkri for useful discussion with him.

5 References

[1] W.K Wootters and W.H.Zurek, Nature 299,802(1982)

[2] A.Ekert, Phys.Rev.Lett. 67 (1991)661

[3] L.M.Duan and G.C.Guo,Phys.Rev.Lett. 80 (1998)4999

[4] V.Buzek and M.Hillery, Phys.Rev.A 54 (1996)1844

[5] D.Bruss, P.D.DiVincenzo, A.Ekert, C.A.Fuchs, C.Macchiavello and J.A.Smolin, Phys.Rev.A 57 (1998)2368

[6] Einstein, Podolsky and Rosen, Phys.Rev. 47 (1935)777

[7] C.H.Bennett and G.Brassard, Proceedings of IEEE International Conference on Computers, System and Signal Processing, Bangalore, India, 1984, pp.175-179

[8] C.H.Bennett and S.J.Weisner, Phys.Rev.Lett.69 (1992)2881

[9] C.H.Bennett, G.Brassard, C.Crepeau,R.Jozsa, A.Peres and W.K.Wootters,Phys.Rev.Lett. 70 (1993)1895

[10] C.H.Bennett, H.J.Bernstein, S.Popescu and B.Schumacher, Phys.Rev.A 53 (1996)2046; D.Deutsch, A.Ekert, R.Jozsa, C.Macchiavello,S.Popescu and A.Sanpera, Phys.Rev.Lett. 77 (1996)2818

[11] V.Buzek, V.Vedral, M.B.Plenio, P.L.Knight and M.Hillery, Phys.Rev.A 55 (1997)3327

[12] S.Bandyopadhyay, G.Kar, Phys.Rev.A 60 (1999)3296

[13] A.Peres, Phys.Rev.Lett. 77 (1996)1413

[14] M.Horodecki, P.Horodecki, R.Horodecki, Phys.Lett.A 223 (1996)1

[15] L.Goldenberg and L.Vaidman, Phys.Rev.Lett. 75 (1995)1239

[16] H.Barnum, C.M.Caves, C.A.Fuchs, R.Jozsa and B.Schumacher, Phys.Rev.Lett. 76 (1996)2818

[17] S.Bose, V.Vedral and P.L.Knight, Phys.Rev.A 57 (1998)822

[18] M.Zukowski, A.Zeilinger, M.A.Horne and A.K.Ekert, Phys.Rev.Lett. 71 (1993)4287

[19] P.W.Shor and J.Preskill, Phys.Rev.Lett. 85 (2000)441

[20] W.K.Wootters, Phys.Rev.Lett. 80 (1998)2245

[21] I.Ghiu, Phys.Rev.A 67 (2003)012323

[22] A.Cabello, Phys.Rev.A 61 (2000)052312

[23] C.Li, H-S Song and L.Zhou, Journal of Optics B: Quantum semiclass. opt. 5 (2003)155

[24] N.J. Cerf, Phys. Rev. Lett. 84, 4497 (2000).

[25] N.J. Cerf, J. Mod. Opt. 47, 187 (2000).

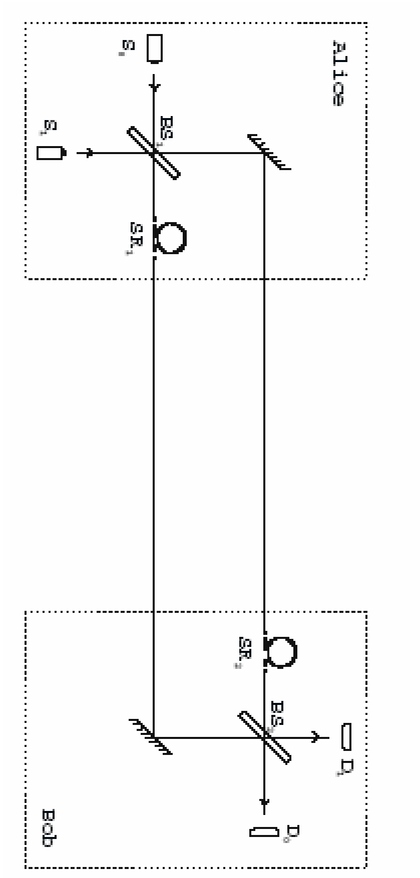

Figure-I. Pictorial representation of the cryptographic scheme is shown in the given figure. The cryptographic scheme based on a Mach-Zehnder interferometer. The device consists of two particle sources S_0 and S_1, a beam-splitter BS_1, two mirrors, two storage rings SR_1 and SR_2, a beam-splitter BS_2 and two detectors D_0 and D_1. The device is tuned in such a way that, if no eavesdropper is present, a particle emitted by S_0 (S_1) is finally detected by D_0 (D_1).

Pictorial representation of the broadcasting of two 2-qubit entangled state between two distant partners, Alice and Bob.

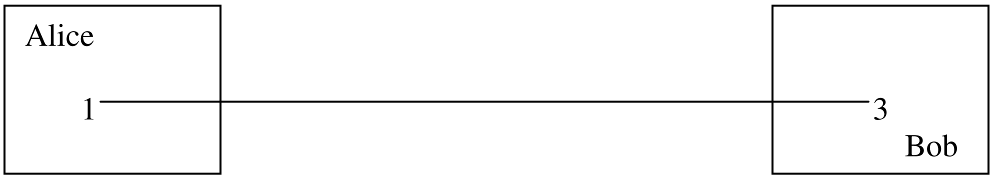

Figure-II. Alice and Bob initially share a two particle entangled state $|\psi\rangle_{13}$.

Figure-III. Alice and Bob operate their local cloning machine on their respective qubits 1 and 3 to produce the copy qubits 2 and 4. Alice and Bob then perform measurement on the cloning machine state vectors and send their measurement result by using the cryptographic scheme shown in figure 1.

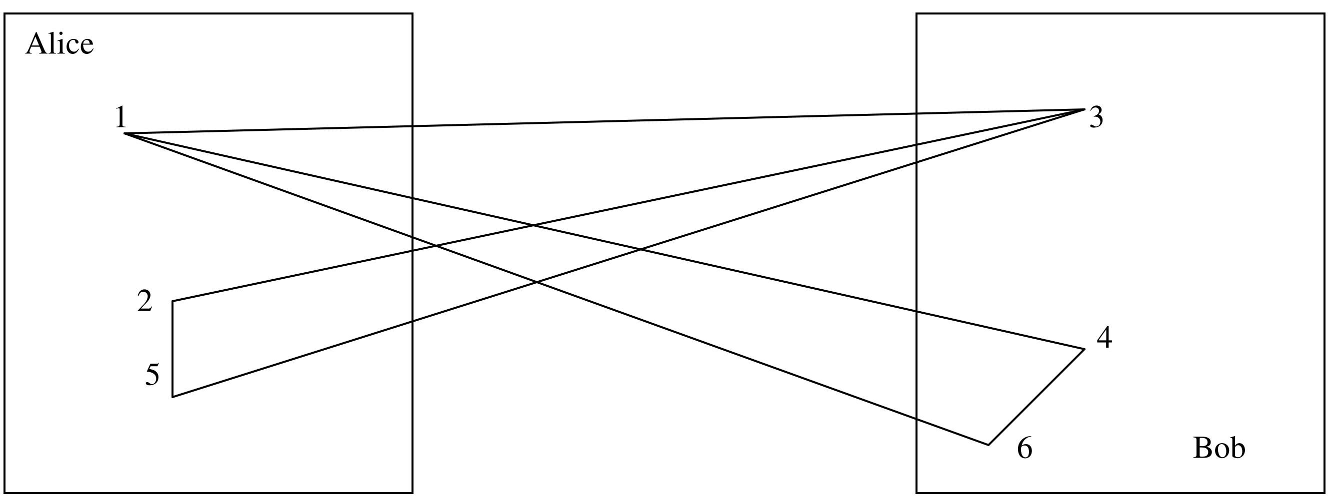

Figure-IV. Alice and Bob then again apply their cloning machine on one of their qubits to produce further copy qubits 5 and 6 respectively. Finally we are able to broadcast two 3-qubit entanglement between two distant partners Alice and Bob for some values of the parameter α^2.

Pictorial representation of the generation of 3-qubit entanglement between three distant partners Alice, Bob and Carol.

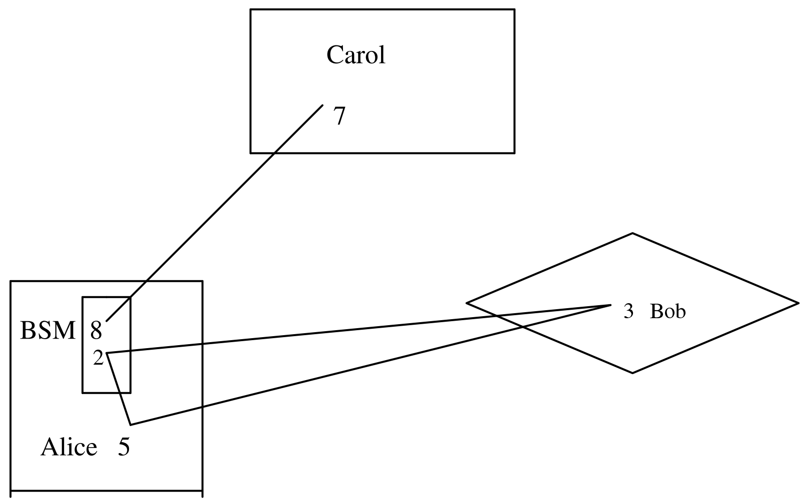

Figure-V. In this figure it is shown that the Bell-state measurement (BSM) performed by Alice on qubits 2 and 8.

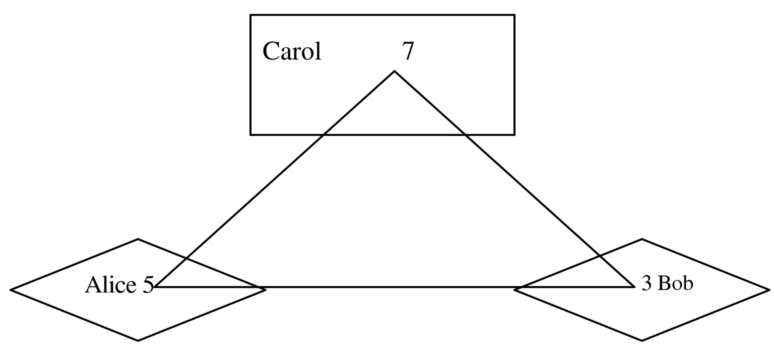

Figure-VI. In this figure, we can see that the three-particle closed entanglement is created. The entanglement is closed in the sense that each party has inter-entanglement between them.

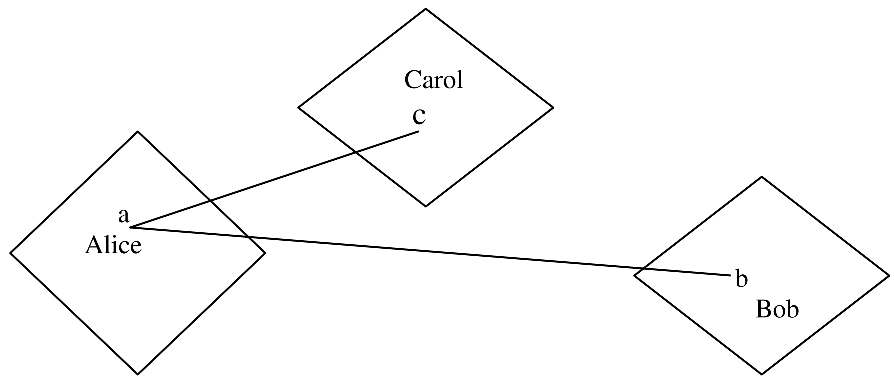

Figure-VII. In this figure, an open entanglement is shown. The entanglement is open in the sense that there is no direct entanglement between the qubits c and b.